1-D diffusion primer

**steady state**

\[ J \propto - \left( \frac{\partial c}{\partial x} \right) \]

\[ J = -D \left( \frac{\partial c}{\partial x} \right) \]

Fick's 1\textsuperscript{st} Law

(also see Fourier's Law)

\[ D \ [\equiv \] \text{cm}^2/\text{s} \]

**non-steady**

\[ \frac{\partial c}{\partial t} = - \frac{\partial J}{\partial x} \]

\[ \frac{\partial J}{\partial x} = \frac{\partial}{\partial x} \left( -D \frac{\partial c}{\partial x} \right) \]

For constant \( D \),

\[ \frac{\partial J}{\partial x} = -D \frac{\partial^2 c}{\partial x^2} \]

\[ \frac{\partial c}{\partial t} = D \frac{\partial^2 c}{\partial x^2} \]

Fick's 2\textsuperscript{nd} Law

**simple scaling**

\[ \frac{\partial c}{\partial t} = D \frac{\partial^2 c}{\partial x^2} \]

\[ \frac{1}{\tau} \sim D \frac{1}{L^2} \]

\[ \tau \sim \frac{L^2}{D} \]

\[ L \sim \sqrt{D\tau} \]

\[ f \sim \frac{D\tau}{L^2} \]
One simple example and solution using the 1-D diffusion equation;

can be used to model outward dispersion of a contaminant layer.

**1-D infinite spike**

\[
C(x, t) = \frac{M}{(4\pi Dt)^{1/2}} \exp\left[-\frac{x^2}{4Dt}\right]
\]

**3-D radial from point source**

\[
C(r, t) = \frac{M}{(4\pi Dt)^{3/2}} \exp\left[-\frac{r^2}{4Dt}\right]
\]
1-D infinite spike

\[ C(x, t) = \frac{M}{(4\pi Dt)^{1/2}} \exp \left[ -\frac{x^2}{4Dt} \right] \]

so \( 2\sigma^2 = 4Dt \)

or \( \sigma^2 = 2Dt \)

and standard deviation \( \sigma \) is like \sqrt{2Dt}

For 1-D infinite spike problem, the characteristic diffusion distance \( X_{cr} = \sqrt{4Dt} \) is where concentration of diffusant is \( 1/e \) of concentration at \( x = 0 \).

Fig. 5.4. Characteristic diffusion distance (the point at which concentration is \( 1/e \) times the concentration at zero distance) as a function of time for various values of the diffusion coefficient \( D \).
characteristic diffusion distance will always be proportional to $\sqrt{Dt}$

$$L \propto \sqrt{Dt}$$  \hspace{1cm} (5.21)

or, bearing in mind that $L$ can take different values depending on the problem,

$$L \sim \sqrt{Dt}$$  \hspace{1cm} (5.22)

Similarly, one can define a characteristic timescale of diffusion $\tau$

$$\tau \sim \frac{L^2}{D}$$  \hspace{1cm} (5.23)

The scaling variables provide useful approximations for describing the progress of diffusion and the approach to equilibrium in space and time in diffusion problems.
normalized concentration, \( c' \)

\[
\zeta = c' = \frac{c - c_2^0}{c_1^0 - c_2^0}
\]

so \( c = c(x, t) = c_2^0 + c' (c_1^0 - c_2^0) \)
normalized concentration

\[ c(x, t) = c_2 + (c_1 - c_2)\zeta \]

\[ \zeta = 1 - \text{erf}\left(\frac{x}{2\sqrt{Dt}}\right) \]
\[ \zeta = \left(\frac{1}{2}\right) \left[ 1 - \text{erf}\left(\frac{x}{2\sqrt{Dt}}\right) \right] \]
\[ \zeta = \left(\frac{1}{2}\right) \left[ \text{erf}\left(\frac{b-x}{2\sqrt{Dt}}\right) + \text{erf}\left(\frac{b+x}{2\sqrt{Dt}}\right) \right] \]

**Fig. 5.5.** Simple diffusion problem solutions. (a) Fixed concentration at \( x = 0 \) that diffuses into the positive \( x \) domain with time, following equation (5.25). (b) Initial normalized concentrations of unity (negative \( x \) domain) and zero (positive \( x \) domain) with step-function separation at \( x = 0 \). Concentrations approach equilibrium by rotating about a fixed point of the mean concentration at \( x = 0 \). (c) Initial normalized concentration of unity between \(-0.5 < x < 0.5\), and zero concentration outside this range.
Excel demonstrations of simple diffusion problems.
The fundamental variable in all these (and all the previous examples) is $Dt/a^2$ (or $Dt/l^2$).
Much of the time, people don’t do full-on analytical solutions, but instead use either numerical models, or analytical approximations (see below).

fractional retention, $F$

fractional loss, $f$

<table>
<thead>
<tr>
<th>Geometry</th>
<th>Equation</th>
<th>Applicable $f$ range</th>
</tr>
</thead>
<tbody>
<tr>
<td>Plane sheet (half-width = $l$)</td>
<td>$f = 1 - \frac{8}{\pi^2} \sum_{n=0}^{\infty} \frac{1}{(2n+1)^2} \exp \left[ \frac{-(2n+1)^2}{4l^2} D t / (4l^2) \right]$</td>
<td>all $f$</td>
</tr>
<tr>
<td></td>
<td>$f \approx \frac{2}{\sqrt{\pi}} \left( \frac{D t}{l^2} \right)^{1/2}$</td>
<td>$f \leq 0.60$</td>
</tr>
<tr>
<td></td>
<td>$f \approx 1 - \frac{8}{\pi^2} \exp \left[ -\frac{\pi^2 D t}{4l^2} \right]$</td>
<td>$f \geq 0.45$</td>
</tr>
<tr>
<td>Cylinder (radius = $a$)</td>
<td>$f = 1 - 4 \sum_{n=1}^{\infty} \frac{1}{a_n^2} \exp \left[ -a_n^2 \frac{D t}{a^2} \right]$</td>
<td>all $f$</td>
</tr>
<tr>
<td></td>
<td>$f = \frac{4}{\sqrt{\pi}} \left( \frac{D t}{a^2} \right)^{1/2} - \frac{D t}{a^2}$</td>
<td>$f \leq 0.60$</td>
</tr>
<tr>
<td></td>
<td>$f \approx 1 - \frac{9}{13} \exp \left[ -5.78 \frac{D t}{a^2} \right]$</td>
<td>$f \geq 0.60$</td>
</tr>
<tr>
<td>Sphere (radius = $a$)</td>
<td>$f = 1 - \frac{6}{\pi^2} \sum_{n=1}^{\infty} \frac{1}{n^2} \exp \left[ -n^2 \frac{\pi^2 D t}{a^2} \right]$</td>
<td>all $f$</td>
</tr>
<tr>
<td></td>
<td>$f \approx 6 \left( \frac{D t}{a^2} \right)^{1/2} - \frac{3 D t}{a^4}$</td>
<td>$f \leq 0.85$</td>
</tr>
<tr>
<td></td>
<td>$f \approx 1 - \frac{6}{\pi^2} \exp \left[ -\frac{\pi^2 D t}{a^2} \right]$</td>
<td>$f \geq 0.85$</td>
</tr>
</tbody>
</table>

should have $\pi^{1/2}$ in the denom of 1st term.
fractional retention, $F$
fractional loss, $f$

**infinite sheet/plane sheet**

![Graph](image)

- $f$, fractional loss
- $D t / a^2$

**Infinite cylinder**

- radius = $a$

**Sphere**

- radius = $a$
temperature dependence of diffusion

\[ D = D_0 \exp \left[ - \frac{E_a + PV_a}{RT} \right] \]

Above: dependence of D on T, P, and three parameters distinctive to the diffusant and medium \((E_a, V_a, D_0)\)

We will simplify this...

Figure 5-5 Schematic illustration of the four possible mechanisms of diffusion transport: exchange, vacancy, interstitial, and interstitialcy.

McDougall & Harrison, 1999
temperature dependence of diffusion

interstitial diffusion: lattice strain imposed by impurity, but strain is higher during movement from one site to another

\[ D = D_0 \exp \left[ -\frac{E_a + PV_a}{RT} \right] \]

at const P, the law of mass action for the migration "reaction," adapted to diffusion could be written

\[ D = A \exp \left[ -\frac{\Delta G}{RT} \right] \]
\[ D = A \exp \left[ -\frac{\Delta H - T\Delta S}{RT} \right] \]
\[ D = A \exp \left[ \frac{\Delta S}{R} \right] \exp \left[ -\frac{\Delta H}{RT} \right] \]

Simplified, important result............

\[ D = D_0 \exp \left[ -\frac{E_a}{RT} \right] \]
Actually, a lot of the time, we can only measure $D/a^2$, not $D$.

- *same a, different T*: low $D$ (red), high $D$ (light blue)
- *same T, different a*: large $a$ (dark blue), small $a$ (light blue)

**Graphs:**
- **4He in titanite**: $\ln(D/a^2)$ vs. $10^4/T$ (K$^{-1}$) for different grain sizes with $T_c$ values.
- **4He in apatite**: $\ln(D/a^2)$ vs. $10^4/T$ (K$^{-1}$), showing a typical uncertainty.

*References:* Reiners & Farley, 1999; Farley, 2000
\[ D = D_0 \exp\left(-\frac{E_a}{RT}\right) \]

\[ \frac{D}{a^2} = \frac{D_0}{a^2} \exp\left[-\frac{E_a}{RT}\right] \]

\[ \ln\left(\frac{D}{a^2}\right) = \ln\left(\frac{D_0}{a^2}\right) + \frac{-E_a}{R} \cdot \frac{1}{T} \]

methods for measuring D (or D/a^2) at different temperatures:
- NRA
- RBS
- LA-ICP-MS
- SIMS
- step-heating
all for $a = 100 \, \mu m$
step-heating experiments

\[ f = 1 - \frac{6}{\pi^2} \sum_{n=1}^{\infty} \frac{1}{n^2} \exp \left[ -n^2 \frac{D t}{a^2} \right] \]

all \( f \)

\[ t \approx \frac{6(Dt)^{1/2}}{\pi^{1/2} \lambda a^2} \quad f \leq 0.85 \]

\[ t \approx 1 - \frac{6}{\pi^2} \exp \left[ -\frac{\pi^2 D t}{a^2} \right] \quad f \geq 0.85 \]

---

**Diagram:**

- **T-t schedule of step-heating experiment:**
  - \( t_i \)
  - \( t_{i+1} \)
  - \( i \text{th step} \)
  - \( (i+1) \text{th step} \)

- **Corresponding \( D/a^2 \) of each step:**
  - \( (D/a^2)_i t_i \)
  - \( (D/a^2)_{i+1} (t_{i+1}-t_i) \)

- **“Lumped” \( Dt/a^2 \) value for all steps:**
  - \( \Sigma(t_{i+1}) \)

---

\[
\left( \frac{D}{a^2} \right)_{i+1} = \frac{-\pi^2}{3} (f_{i+1}-f_i) - 2\pi \left( \sqrt{1 - \frac{\pi}{3} f_{i+1}} - \sqrt{1 - \frac{\pi}{3} f_i} \right) \]

(5.33)

for \( f \leq 0.85 \), and

\[
\left( \frac{D}{a^2} \right)_{i+1} = \frac{\ln \left[ \frac{1-f_i}{1-f_{i+1}} \right]}{\pi^2 (t_{i+1}-t_i)} \]

(5.34)

for \( f \geq 0.85 \). *Fechtig and Kalbitzer [1966]*
Fig. 5.8. Simulated Arrhenius plots for step-heating release experiments involving all prograde steps (a) and cycled prograde and retrograde steps (b) all of 3600 s durations for samples with $E_a$ of 165 kJ/mol, $D_0$ of 0.01 cm$^2$/s, and comprising mixtures of four discrete domain sizes of varying size (labeled diagonal lines). Gray fill represents a mixture with equal proportions of the domains, and the white and black filled symbols represent higher proportions of smaller and larger domains, respectively.
the log $r/r_0$ plot

$$\ln \left( \frac{D}{a^2} \right)_0 - \ln \left( \frac{D}{a^2} \right)$$

Assuming identical $E_a$ for the steps/domains yields

$$\ln \left[ \left( \frac{D_0}{a^2} \right)_0 \left( \frac{a^2}{D_0} \right) \right]$$

**Fig. 5.9.** The $\ln(a/a_0)$ (also called log($r/r_0$) plot in many applications). This index is one half the difference in the vertical displacement (apparent difference in $\ln(D/a^2)$) of successive steps in the step-heating experiments shown in Fig. 5.8. This is used to constrain the distribution and proportions of domain sizes in step-heating results measured on multidomain samples. These $\ln(a/a_0)$ trends are not sensitive to heating schedule, and are the same for both examples shown in Fig. 5.8. Dashed horizontal lines represent tenfold differences in domain sizes.
three "endmember" interpretations of a thermochronologic "age"

**Fig. 3** Schematic illustration of three “end-member” ways that thermochronologic ages may be interpreted. Left panel: Closure, in which cooling from high to low temperature and subsequent accumulation of daughter product in a one-to-one correlation with time leads to the concept of a characteristic temperature $T_c$. $T_c$ depends mostly on the kinetic parameters of the daughter product loss (but also on cooling rate and other assumptions), that can be associated with the thermochronometer’s age. Middle: Thermochronometers held at any temperature will eventually experience partial retention of daughter product and approach an equilibrium age $t_{eq}$, representing a balance of daughter product growth and loss. $t_{eq}$ depends on the thermochronometer’s kinetic parameters and isothermal holding temperature. Thermochronometers will approach $t_{eq}$ faster for higher temperatures and younger $t_{eq}$. Right: Thermochronometers residing at low temperature and approximating complete daughter retention may be partially or fully reset by short transient heating events. With some assumptions, including timing of the event, combining two or more thermochronometers with different kinetics can constrain the duration and temperature of the resetting event.
Diffusion/production in a sphere:

$$\frac{\partial^4 He(r,t)}{\partial t} = D(t) \left[ \frac{\partial^2}{\partial r^2} \frac{\partial^4 He(r,t)}{\partial r^2} + \frac{2}{r} \frac{\partial^4 He(r,t)}{\partial r} \right] + P(t)$$

(after Wolf et al., 1998)

The production term:

$$P(t) = 8\lambda_{238}^{238} U(t) + 7\lambda_{235}^{235} U(t) + 6\lambda_{232}^{232} Th(t) + \lambda_{147}^{147} Sm(t)$$

The diffusivity term:

$$\frac{D(t)}{a^2} = \frac{D_0}{a^2} e^{-E_a/RT(t)}$$

General solution (Wolf et al., 1998)

$$\frac{4 He}{P} = t' = \frac{a^2}{D} \left[ \frac{1}{15} - \sum_{n=1}^{\infty} \frac{6}{\pi^4 n^4} \exp\left(-n^2 \pi^2 D \frac{a^2}{t}\right) \right] + \frac{4 He^*}{P} \sum_{n=1}^{\infty} \frac{6}{\pi^2 n^2} \exp\left(-n^2 \pi^2 D \frac{a^2}{t}\right)$$

(age)
General solution *(Wolf et al., 1998)*

\[
\frac{^4\text{He}}{P} = t' = \frac{a^2}{D} \left[ \frac{1}{15} - \sum_{n=1}^{\infty} \frac{6}{\pi^4 n^4} \exp \left( -n^2 \pi^2 \frac{D}{a^2} t \right) \right] + \frac{^4\text{He}^*}{P} \sum_{n=1}^{\infty} \frac{6}{\pi^2 n^2} \exp \left( -n^2 \pi^2 \frac{D}{a^2} t \right)
\]

2. Closure perspective:

\[
\frac{^4\text{He}}{P} = t' = \frac{a^2}{D} \left[ \frac{1}{15} - \sum_{n=1}^{\infty} \frac{6}{\pi^4 n^4} \exp \left( -n^2 \pi^2 \frac{D}{a^2} t \right) \right]
\]
General solution (Wolf et al., 1998)

\[
\frac{4\text{He}}{P} = t' = \frac{a^2}{D} \left[ \frac{1}{15} - \sum_{n=1}^{\infty} \frac{6}{\pi^4 n^4} \exp \left( -n^2 \pi^2 \frac{D}{a^2} t \right) \right] + \frac{4\text{He}^*}{P} \sum_{n=1}^{\infty} \frac{6}{\pi^2 n^2} \exp \left( -n^2 \pi^2 \frac{D}{a^2} t \right)
\]

1. Equilibrium perspective:

\[
\frac{4\text{He}}{P} = t' = \frac{a^2}{D} \frac{1}{15}
\]
General solution (Wolf et al., 1998)

\[ \frac{^4\text{He}}{P} = t' = \frac{a^2}{D} \left[ \frac{1}{15} - \sum_{n=1}^{\infty} \frac{6}{\pi^4 n^4} \text{exp}\left( -n^2 \pi^2 \frac{D}{a^2} t \right) \right] + \frac{^4\text{He}^*}{P} \sum_{n=1}^{\infty} \frac{6}{\pi^2 n^2} \text{exp}\left( -n^2 \pi^2 \frac{D}{a^2} t \right) \]

3. Resetting perspective:

\[ \frac{^4\text{He}}{P} = t' = \frac{^4\text{He}^*}{P} \sum_{n=1}^{\infty} \frac{6}{\pi^2 n^2} \text{exp}\left( -n^2 \pi^2 \frac{D}{a^2} t \right) \]
equilibrium ages and approaches to them

equilibrium age

t' = t_{eq} = \frac{a^2}{D} \left( \frac{1}{15} \right)

t^* \approx \frac{a^2}{D} \left( \frac{1}{\pi^2} \right)

and the $t_{eq}$ is essentially reached within 5 $t^*$. 
equilibrium ages as expressed in a partial retention zone (PRZ)

Fig. 5.13. Partial retention zones and apparent dates for the apatite and zircon (U-Th)/He systems for isothermal holding at temperatures shown on y-axis, for durations of 1, 3, 10, 30, 100, 300, 1000, and 3000 Ma. The equilibrium date ($t_{eq}$, equation 5.46) is reached quickly for lower temperature samples, resulting in a steep depth-date trend at shallow depths, but the $t_{eq}$ requires much longer ingrowth times for higher temperatures and older dates, at greater depth, resulting in a shallow temperature-date slope. Temperature is plotted increasing downwards to correspond with the typical situation of increasing temperature with depth in the crust. The partial retention zone (PRZ) can be defined as the range of temperatures where $0.1 < t'/t_{eq} < 0.9$, and these limits for each isothermal holding duration are shown as filled circles. Note that both the upper and lower temperature limits of the PRZ decrease with time. The apatite system assumes Durango apatite kinetics ($E_a = 138$ kJ/mol; $D_0 = 31.6$ cm$^2$/s; Farley [2000]) and the zircon system assumes typical kinetics ($E_a = 168$ kJ/mol; $D_0 = 0.46$ cm$^2$/s; Reiners et al. [2004]); both assume a spherical domain with radius of 50 μm.
If duration of resetting event is short compared with half-life of parent,

\[
\frac{N}{P} = t' = \frac{N}{P} \sum_{n=1}^{\infty} \frac{6}{\pi^2 n^2} \exp\left(-n^2 \pi^2 \frac{D}{a^2} t\right)
\]
How do we invert these partially reset ages for $T$’s and $t$’s?
for non-square pulse $t$-$T$ history:

$$\tau(T,t) = \frac{D_0}{a^2} \int_0^t \exp \left[ -\frac{E_a}{R \cdot T(t')} \right] dt'$$
\[ \tau_1(T,t) = \frac{D_{01}}{a_1^2} \exp \left[ \frac{-E_{a1}}{R \cdot T(t)} \right] \int_0^t dt \]

\[ \tau_2(T,t) = \frac{D_{02}}{a_2^2} \exp \left[ \frac{-E_{a2}}{R \cdot T(t)} \right] \int_0^t dt \]

**arbitrary t-T path**

\[ (Dt/a^2)_1 = \frac{D_{01}}{a_1^2} \exp \left[ \frac{-E_{a1}}{R \cdot T} \right] t \]

\[ (Dt/a^2)_2 = \frac{D_{02}}{a_2^2} \exp \left[ \frac{-E_{a2}}{R \cdot T} \right] t \]

**square-pulse equivalent**

\[ T_{1-2} = \frac{E_{a2} - E_{a1}}{R} \ln \left( \frac{\tau_1 D_{02} a_1^2}{\tau_2 D_{01} a_2^2} \right) \]

\[ t_{1-2} = \frac{a_1^2}{D_{01}} \exp \left[ \frac{E_{a1}}{R \cdot T} \right] \tau_1 \]
Fractional-loss for Ar and He in ALH84001

- **apatite He**
  - $E_a = 138 \text{ kJ/mol}$
  - $D_0 = 31.6 \text{ cm}^2/\text{s}$
  - $a = 27 \mu\text{m}$
  - $T_c = 59 \degree\text{C}$

- **maskelynite Ar**
  - $E_a = 76.8 \text{ kJ/mol}$
  - $D_0/a^2 = 0.15 \text{ s}^{-1}$
  - $T_c = 6 \degree\text{C}$

The graph shows the relationship between $\ln(t)$ and $10^4/T$ for different materials and fractional losses. The parameters for each material are indicated on the graph.
Closure age

Closure temperature $T_c$

In strictest sense requires $T$ to decrease linearly as a function of $1/t$...

...and leads to this important variable...

$$\tau = \frac{-RT^2}{(E_a dT/dt)}$$

(not the same tau we were just talking about)

Closure temperature (mathematically)

$$T_c = \frac{\left( \frac{E_a}{R} \right)}{\ln \left[ \frac{ART_c^2 \left( D_0/a^2 \right)}{E_a \left( dT/dt \right)} \right]}$$

Dodson, 1973
Fig. 5.18. Closure temperatures for various thermochronometers as a function of cooling rate. Kinetic parameters are as compiled in Reiners [2009].
Fig. 5.19. Variations in closure temperature ($T_c$) as a function of $E_a$, $a$, $dT/dt$, $D_0$, and domain geometry. Default values (fraction of default values = 1) for example are $E_a = 169$ kJ/mol, $D_0 = 0.5$ cm$^2$/s, $a = 60$ mm, and $dT/dt = -10$ K/Ma, which yield a $T_c$ of 182 °C. $T_c$ for variations in $E_a$ is shown on right $y$-axis; all other variations are shown on left $y$-axis. Fractional variations in $E_a$ have by far the strongest control on $T_c$. What controls closure temperatures?
closure profiles

\[
\frac{E_a}{RT_c(x)} = \ln \left( \frac{\tau D_0}{a^2} \right) + G(x)
\]

\[
\tau = \frac{-RT^2}{(E_a dT/dt)}
\]

Table 5.2 Spatially resolved values for G(x), the closure function, where x is the fractional spatial coordinate from the core of a diffusion domain, for the three primary domain geometries

<table>
<thead>
<tr>
<th>x</th>
<th>Plane sheet</th>
<th>Cylinder</th>
<th>Sphere</th>
</tr>
</thead>
<tbody>
<tr>
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<tr>
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Fig. 5.22. Theoretical T_c profiles (solid lines) and integrated T_c profiles (dotted lines) for the titanite (U-Th)/He system, calculated by the method of Dodson [1986], assuming spherical grains with 150 μm radius. Integrated T_c profiles are He T_c values of a portion of the crystal from core to rim at that radial distance. These profiles do not include the effects of alpha ejection on the diffusive profiles.

Source: Dodson [1986]. Reproduced with permission of Trans Tech Publications, Ltd.
$^{40}\text{Ar}/^{39}\text{Ar}$ ages in phlogopite crystals from kimberlite xenoliths

Kelley and Wartho, 2000
Eclogitization of lower crustal granulites by fluid migration through shear zones

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The granulite facies assemblages of the anorthisitic rocks of the Bergen Ares (stable at 800–900°C and 10 kbar) have been transformed to eclogite facies assemblages (stable at 700–750°C and 16–19 kbar) in the vicinity of Caledonian shear zones. This section of the root zone of the Caledonian mountain chain reveals a deep polymetamorphic crust where Precambrian granulites (mean density 3.02 g/cm³) and Caledonian eclogites (mean density 3.19 g/cm³) alternate on a scale of meters over a minimum area of 3 × 12 km. Detailed mapping of three localities shows that eclogites account for up to 30–45% of the rock volume. The stabilization of the eclogite mineralogy is controlled by fluids penetrating these deep crustal shear zones. The eclogitization is independent of preexisting compositional variation in this anorthisite-norite complex. The Bergen Ares example suggests that the amount of eclogite versus granulites in the lowermost crust is a function of deformation and fluid access, rather than being controlled by T, P and rock composition alone. These relationships may explain the gradual increase in seismic velocity observed in some deep crustal sections and also the complex reflection pattern obtained from the lowermost crust in many areas.
Eclogite: >15 kbar, ~700 °C, ~450 Ma

...but, phlogopite outside eclogite zones has Rb/Sr age of 830-880 Ma...

and the phlog Rb/Sr $T_c$ is ~ 300-550 °C

Hmm...this rock was at 700 °C at 450 Ma, but its Rb/Sr phlogopite age ($T_c$ hundreds of degrees lower) is 850 Ma...
### Rb-Sr mica ages that are “too-old” or “pre-metamorphic”

<table>
<thead>
<tr>
<th>Location</th>
<th>References</th>
</tr>
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<tr>
<td>Western Norway</td>
<td>Verschure et al., 1980; Kuhn et al., 2000; Glodny et al., 2007</td>
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<td>Central &amp; Eastern Alps</td>
<td>von Blackenburg et al., 1989; Steck &amp; Hunziker, 1994</td>
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<td>Urals</td>
<td>Glodny et al., 2003</td>
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<td>Front Range</td>
<td>Patel et al., 1999</td>
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### Suggested causes:

1. diffusion kinetics are incorrect
2. modal control on Sr partitioning
3. “shielding” by surrounding phases
4. $P$ effect on kinetics
5. fluids/recrystallization required

“Rb/Sr mineral systematics reflect magmatic crystallization despite of [sic] later metamorphic temperatures of 650 °C. The Rb/Sr mineral data provide hygrochronological rather than thermochronological constraints.

“For fluid-absent rocks, with the possible exception to ultrahigh temperature rocks like granulites, the classical theory of cooling ages does not apply.”

*Glodny et al., 2003*
$f = 0.99$

Contours

$\ln(t) = \ln\left( f, \frac{a^2}{D_0} + \frac{E_a}{R} \left( \frac{1}{T} \right) \right)$
same rocks,
same eclogitic shear zones

excess Ar uptake profiles in phlogopite and amphibole

invert for $T$ and duration of heating...
What are the characteristic lengthscale and timescale of a metamorphic event?
Brief thermal pulses during mountain building recorded by Sr diffusion in apatite and multicomponent diffusion in garnet

Jay J. Ague, Ethan F. Baxter

the classic Barrovian metamorphic zones

have diffusion profiles

requiring peak P-T conditions \( \sim 10^5 \) yr

![Graph showing Sr wt. % vs. X (um) with best-fit models A, B, C and Sr diffusion profiles for models A, B, C over time.](image)