• Board stuff

• Decay and growth equations
  • *deriving fundamental equations for growth and decay, half-life*
  • *growth curves as a function of time, parent-daughter ratios*
  • *normalization to stable/nonradiogenic isotope to get isochron equation*
• Board stuff
  • Trace elements
    • *parent-daughter ratio fractionation*
    • *Henry’s Law and trace elements*
    • *Fractionation by batch processes*
    • *derivation of mass balance equations*
    • *simple-mixing equations*
Nuclear Drop Model (Gamow, 1928; Wiegzacker, 1935)

\[ E = (15.8 \text{ MeV}) \times A - (17.8 \text{ MeV}) \times A^{2/3} - (0.71 \text{ MeV}) \times \frac{Z^2}{A^{1/3}} - (23.7 \text{ MeV}) \times \frac{(A-2Z)^2}{A} \pm \frac{11.2 \text{ MeV}}{A^{1/2}} \]

For a given \( A \) and varying \( Z \), binding energy defines a parabola describing balance of coulombic repulsion and strong nuclear force.

![Graph showing the relationship between nuclear energy and charge number (Z) for isotopes with mass 87.](image)

Fig. 2.4. Excess nuclear energy, in MeV, for the addition or subtraction of protons from the stable isotope at mass 87, \(^{87}\text{Sr}\). Each isotope is labeled along the curve with its radioactive decay half-life.

little energy released from \(^{87}\text{Rb}\)-\(^{87}\text{Sr}\) decay
Table 6.1 Rb–Sr isotopic composition in atom% 

<table>
<thead>
<tr>
<th>Element</th>
<th>Mass</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rb</td>
<td>84</td>
</tr>
<tr>
<td>Sr</td>
<td>85</td>
</tr>
<tr>
<td></td>
<td>86</td>
</tr>
<tr>
<td></td>
<td>87</td>
</tr>
<tr>
<td></td>
<td>88</td>
</tr>
</tbody>
</table>

\[ ^{87}\text{Rb} \rightarrow ^{87}\text{Sr} \]

\[ t_{1/2} = 48 \text{ Ga} \]

\[^{87}\text{Sr}/^{86}\text{Sr} \text{ ratio of 0.705.} \]
Rb → Sr

\[ \frac{87\text{Sr}}{86\text{Sr}} = \left(\frac{87\text{Sr}}{86\text{Sr}}\right)_i + \frac{87\text{Rb}}{86\text{Sr}}(e^{\lambda t} - 1) \]

t_{1/2} = 48 \text{ Ga}

Fig. 3.2. Schematic Rb–Sr isochron diagram for a suite of co-magmatic igneous minerals.
Isochron assumptions

1. closed system since crystallization (or, only changes in isotope ratios are the result of radioactive decay)
2. within analytical error, all phases formed at the same time
3. all phases started with the same initial ratio
4. we know $\lambda$ precisely and accurately
5. good analyses

![Isochron diagram](image)

Fig. 3.2. Schematic Rb–Sr isochron diagram for a suite of co-magmatic igneous minerals.
Rb-Sr Isochron to Determine Age of NWA 7034

Age = 2089 ± 81 Ma
Initial \( \frac{^{87}Sr}{^{86}Sr} \) = 0.71359 ± 54
MSWD = 6.6

Fig. 7. Rb–Sr isochron diagram for minerals of the contact metamorphosed pelite sample SP1 from Bluegrass Creek. The $1420 \pm 43$ Ma isochron is defined by biotite, whole rock and feldspar, and has an MSWD of 1.56.
Figure 10.10 Whole-rock Rb-Sr isochron of the Salisbury pluton from Rowan County, North Carolina. The initial $^{87}\text{Sr}/^{86}\text{Sr}$ ratio of this intrusive is well within the range of isotopic compositions of strontium in the upper mantle as outlined in Figure 10.3. Consequently the conclusion is justified that the magma from which the Salisbury pluton crystallized evolved from a parent magma which originated in the upper mantle and was not contaminated with foreign strontium from the continental crust. The initial $^{87}\text{Sr}/^{86}\text{Sr}$ ratio has been increased by +0.0006 to make it compatible with an $^{87}\text{Sr}/^{86}\text{Sr}$ ratio of 0.7080 for the Eimer and Amend strontium isotope standard. (Fullagar et al., 1971.)

Figure 10.9 Whole-rock Rb-Sr isochron of biotite-bearing quartz monzonites from the Martin Dome of the Miller Range in the Transantarctic Mountains. The high initial $^{87}\text{Sr}/^{86}\text{Sr}$ ratio indicates that this is an example of a granitic intrusive that formed from magma which either assimilated large amounts of old sialic rocks or was produced by partial melting of the granitic basement rocks of this area. The date has been recalculated to

$$\lambda(^{87}\text{Rb}) = 1.42 \times 10^{-11} \text{y}^{-1}$$

(Gunner, 1974).
Figure 10.11 Contour diagram showing the regional variation of initial $^{87}\text{Sr}/^{86}\text{Sr}$ ratios of volcanic and plutonic rocks of Mesozoic age in California. (Reprinted from Figure 2 (p. 3493) of Kistler, R. W., and Z. E. Peterman (1973) Geological Society of America Bulletin, 84, No. 11, 3489–3512 with permission of G.S.A.)
Meteorites

- irons
- stony-irons
- achondrites
- chondrites
  - ordinary
  - carbonaceous
  - enstatite

Chunks of planetesimals (or planets)
Fig. 3.10. Rb–Sr isochron diagram for whole-rock samples of basaltic achondrites, showing the determination of ‘BABI’. After Papanastassiou and Wasserburg (1969).
Whole-rock Rb–Sr isochron of 10 chondrites of the LL class. The date is relative to $\lambda = 1.42 \times 10^{-11} \text{ yr}^{-1}$ for the decay constant of $^{87}\text{Rb}$ and the initial ratio was adjusted to an $^{87}\text{Sr}/^{86}\text{Sr}$ ratio of 0.71025 for the NBS 987 Sr isotope standard. Data from Minster and Allègre (1981).
Fig. 3.11. Plot of initial Sr isotope compositions for selected meteorites against model ages for condensation or differentiation–metamorphism, based on assumed Rb/Sr ratios in major reservoirs. ADOR = Angra dos Reis. After Gray et al. (1973).
Ages of shergotiites, basaltic to ultramafic achondrites from Mars that compose about three-quarters of all Martian meteorites.

**FIGURE 5.5** Rb–Sr mineral isochrons of the SNC meteorites Shergotty, Zagami, Nakhla, ALH 77005, and QUE 94201 compared to the isochron defined by meteorites from the asteroidal belt. Note that the Rb–Sr dates of all of the martian meteorites are lower than those of most meteorites and their initial \(^{87}\text{Sr}/^{86}\text{Sr}\) ratios are higher than BABI. Data from Papanastassiou and Wasserburg, (1974), Gale et al. (1975), Nyquist et al. (1979), Shih et al. (1982), Jagoutz and Wänke (1986), and Borg et al. (1997).
FIGURE 5.10 Evolution of strontium in three whole rocks (R1, R2, and R3) and in the minerals of R2 (M2). The strontium in the minerals was isotopically homogenized by an episode of thermal metamorphism of short duration. The slope of the whole-rock isochron corresponds to $t_i$, the time elapsed since crystallization of these rocks. The slope of the mineral isochron indicates $t_m$, the time elapsed since the end of the thermal metamorphism.
FIGURE 5.11 Whole-rock and mineral isochrons of granitic rocks from the Carn Chuinneag complex in the Highlands of northern Scotland. The date derived from the whole-rock isochron is the crystallization age of the rocks. The minerals of whole-rock sample 20782 define a separate isochron that dates an episode of thermal metamorphism. Additional age determinations of this complex were reported by Pidgeon and Johnson (1974). Data from Long (1964).
Fig. 1. Generalized geological map of the Laramie Mountains, showing the area of Archean high-grade gneiss and supracrustal rocks in the central Laramie Mountains bounded on the north by the Laramie Peak shear zone and on the south by the 1.43 Ga Laramie Anorthosite Complex. Regional metamorphism of the central Laramie Mountains is associated with collision of Proterozoic island arc terranes against the Archean Wyoming province along the Cheyenne belt.
Fig. 2. Geological sketch map of the central Laramie Mountains, modified from Snyder (1984), showing the location of Elmers rock greenstone belt and Morton Pass, the areas from which samples of pelitic schist were collected. GMP3.5 and GMP4.7 are samples of pelitic schist and metabasite, respectively. SP11 and SP12 are pelitic schist samples from the contact aureole at Morton Pass. Samples SP1–9 are from the area outlined by the box; an enlargement of this area is shown in Fig. 3.

Fig. 3. Enlargement of the map area from which contact metamorphosed pelitic schist samples SP1–9 were collected, showing the contact metamorphic zones defined by Snyder et al. (in press). K, kyanite; S, sillimanite; A, andalusite; Ksp, K-feldspar; C, cordierite; O, orthopyroxene; G, garnet.
Fig. 4. Sm–Nd isochron diagram for whole-rock samples from contact aureole (○) and regional metamorphic terrane (●).

Fig. 5. Rb–Sr isochron diagram for whole-rock samples from contact aureole (○) and regional metamorphic terrane (●). Samples scatter about a slope corresponding to an age of 1780 Ma, the time of regional metamorphism. Reference isochron slopes corresponding to 2640 Ma, the time of deposition of the supracrustal rocks, and to 1430 Ma, the time of contact metamorphism, are shown for comparison.
Fig. 7. Rb–Sr isochron diagram for minerals of the contact metamorphosed pelite sample SP1 from Bluegrass Creek. The 1420±43 Ma isochron is defined by biotite, whole rock and feldspar, and has an MSWD of 1.56.

Fig. 6. Rb–Sr isochron diagram for minerals of the regional metamorphosed pelite sample SP15. The 1450±40 Ma isochron is defined by biotite, whole rock and quartz+feldspar, and has an MSWD of 0. The muscovite–whole rock slope corresponds to a date of 1670 Ma.
WR Sm/Nd isochron: ~2.6 Ga
WR Rb/Sr isochron: ~1.7 Ga
mineral Rb/Sr isochron: ~1.4 Ga

Patel et al., 1999
To first order, these results suggest isotopic resetting (by diffusive transport) at the lengthscale of minerals (~1 cm), but no resetting on lengthscales greater than this. Is this reasonable?

What is timescale of this metamorphic event?

What is lengthscale of expected resetting?

\[ \kappa \sim 25 \text{ km}^2/\text{Ma} \]

\[ \text{D}_{\text{Sr}} \text{ in minerals: } \sim 10^{-15} \text{ cm}^2/\text{s} \]

Patel et al., 1999
Sr isotopes in seawater

- residence time = amount in reservoir / flux in or out of reservoir (at steady state)
- residence time = average amount of time a molecule or atom spends in the reservoir
- residence time of Sr in ocean is ~5 Ma
- mixing time of ocean is a few ka
- Is $^{87}\text{Sr}/^{86}\text{Sr}$ of ocean homogeneous from place to place?
Sr isotopic composition of seawater as a function of time

what causes these variations?

Vezier et al., 1999
TABLE 1

Seawater Sr balance parameters from Palmer and Edmond [14]. Values used in this study that differ from those of Palmer and Edmond are given in parentheses

<table>
<thead>
<tr>
<th>Total Sr in ocean:</th>
<th>$N = 1.25 \times 10^{17}$ (mol)</th>
</tr>
</thead>
<tbody>
<tr>
<td>River input:</td>
<td></td>
</tr>
<tr>
<td>flux</td>
<td>$J_r = 3.3 \times 10^{10}$ (mol/yr)</td>
</tr>
<tr>
<td>isotopic ratio</td>
<td>$R_r = 0.711$</td>
</tr>
<tr>
<td>Hydrothermal alteration:</td>
<td></td>
</tr>
<tr>
<td>flux</td>
<td>$J_h = 1.0 \times 10^{10}$ (mol/yr)</td>
</tr>
<tr>
<td></td>
<td>$(0.82 \times 10^{10})$</td>
</tr>
<tr>
<td>isotopic ratio</td>
<td>$R_h = 0.7035$</td>
</tr>
<tr>
<td></td>
<td>$(0.7030)$</td>
</tr>
<tr>
<td>Diagenesis:</td>
<td></td>
</tr>
<tr>
<td>flux</td>
<td>$J_d = 0.3 \times 10^{10}$ (mol/yr)</td>
</tr>
<tr>
<td></td>
<td>$(0)$</td>
</tr>
<tr>
<td>isotopic ratio</td>
<td>$R_d = 0.7084$</td>
</tr>
</tbody>
</table>

$$N \frac{dR_{sw}}{dt} = J_r (R_r - R_{sw}) + J_h (R_h - R_{sw})$$
Fig. 1. High-resolution seawater Sr isotopic data for the last 100 m.y. Data sources: DSDP 590B and 575 from Richter and DePaolo [3]; DSDP 522, Contessa and DPI-1985 from DePaolo and Ingram [2]; DSDP 305 from Hess et al. [4].

Fig. 2. (a) Sr isotope evolution curve for seawater used in models of the Sr budget of the oceans over the last 100 m.y. (taken from data shown in Fig. 1). (b) Rate of change of the Sr isotopic ratio of seawater taken from (a). Units are change in the 5th decimal place per million years.
Left: how seawater $^{87}\text{Sr}/^{86}\text{Sr}$ would change if only hydrothermal flux changed (and followed apparent seafloor spreading rates)

Left: how Sr flux from rivers and hydrothermal sources would have to change, assuming constant isotopic compositions of each, to get observed seawater $^{87}\text{Sr}/^{86}\text{Sr}$ evolution

Left: $^{87}\text{Sr}/^{86}\text{Sr}$ of rivers required to match observations, if Sr flux stayed constant (at today’s value)

Fig. 3. (a) Calculated seawater Sr isotopic evolution (dots) assuming the hydrothermal flux changes in proportion to the rate of new seafloor generation (dots in Fig. 3b), all other quantities being held at their present-day value (Table 1). The measured Sr isotopic evolution curve is given by the heavy solid curve. (b) Circles are the riverine Sr flux (in units of $10^{11}$ mol/yr) required by a model calculation that reproduces the measured seawater Sr isotopic evolution curve when the average $^{87}\text{Sr}/^{86}\text{Sr}$ of river water is held at its modern value of 0.711, and the hydrothermal flux (with $^{87}\text{Sr}/^{86}\text{Sr} = 0.703$) changes with time in proportion to the rate of new seafloor generation as shown by the dots. (c) Change in the average $^{87}\text{Sr}/^{86}\text{Sr}$ of the riverine flux needed to reproduce the measured seawater Sr isotopic evolution curve when the riverine Sr flux is held at its present-day value ($3.3 \times 10^{11}$ mol/yr, $^{87}\text{Sr}/^{86}\text{Sr} = 0.711$) and the hydrothermal flux varies as shown in (b).
Below: how erosion rate would have to change in the Himalayas in order to explain the seawater $^{87}\text{Sr}/^{86}\text{Sr}$ curve.

Fig. 8. The erosion rate of the Himalayan–Tibetan Plateau as a function of time that provides the amount of dissolved Sr needed to account for the rise in seawater $^{87}\text{Sr}/^{86}\text{Sr}$ over the past 50 m.y.

Below: inferred exhumation history of (part of) the Himalayas, from thermochronology

Fig. 5. Unroofing of the Ou Xu pluton of the Gandgese belt shown in terms of the changing depth of minerals inferred from $^{40}\text{Ar}/^{39}\text{Ar}$ thermochronometry (from Richter et al. [13]). The heavy solid curve is for a single feldspar sample originally 11 km below the surface at the time of emplacement, while the dashed curve was constructed from data by Copeland et al. [12], who give the age and closure temperature (from which we can infer a depth–age) of minerals sampled at different present elevations. The circles show the depth–age of the individual samples analyzed by Copeland et al. [12] after adjustment to a reference elevation that corresponds to each sample having originated at a depth of 12 km. Both unroofing curves indicate a period of exceptionally rapid erosion beginning at about 20 Ma, which is the same time period during which the most rapid increase in the $^{87}\text{Sr}/^{86}\text{Sr}$ of seawater occurred (see Fig. 2), and suggests a causal relationship.